Name:

MVMSC Monthly Math Contest

Time: Whole lunch period.

Directions: No calculators. For answers expressible in multiple ways, any is allowed, but fractions must be simplified and denominators rationalized. Your score is the sum of the point values of problems you solve.

#	Question	Answer
1 (1)	What is the largest positive integer that can't be expressed as the sum of positive multiples of 5 and 7?	
2 (1)	How many numbers between 1000 and 9999 have the property that their digits are in strictly increasing order?	
3 (2)	A positive integer has 9 positive factors. What is the sum of all possible numbers of factors its cube can have?	
4 (2)	A solid cube has side length 9 inches. A 8-inch by 8-inch square hole is cut into the center of each face. The edges of each cut are parallel to the edges of the cube, and each hole goes all the way through the cube. What is the volume, in cubic inches, of the remaining solid?	
5 (2)	Triangle ABC has $AC = 75$ and $BC = 50$. Points K and L are located on $\overline{AC}$ and $\overline{AB}$ respectively so that $AK = CK$, and $\overline{CL}$ is the angle bisector of angle C. Let P be the point of intersection of $\overline{BK}$ and $\overline{CL}$, and let M be the point on line BK for which K is the midpoint of $\overline{PM}$. If $AM = 30$, find LP.	
6 (3)	Suppose that x, y, and z are three positive numbers that satisfy the equations $xyz = 1$, $x + \frac{1}{z} = 13$, and $y + \frac{1}{x} = 17$. Find $z + \frac{1}{y}$.	
7 (3)	Triangle ABC has side lengths $AB = 5, BC = 7, CA = 6$. Let ω be the circumcircle of ABC and I be the incenter of ABC. Line AI intersects ω again at point X. Find the length of IX.	